A review of turbomachinery tip gap effects

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Part 2: Rotating machinery

This two part review covers experiments examining the effects of blade tip gaps encountered in turbomachines and the methods by which the synthesised data are currently used in turbomachine design and analysis. Data gained since the 1930's are subdivided for convenience into cascade[1] (Part 1) and rotating machinery (Part 2) data, with a further subdivision into diffusing, or compressor type flows and accelerating, or turbine type flows. The overall trend is that an increasing tip gap, whose effect can reach over most or all of the blade height, reduces turbomachine performance. There is some evidence among the compressor and compressor cascade data that an optimum gap exists when the opposing effects of secondary flows and tip leakage with rotor/wall relative movement tend to balance. Turbine data are, in general, more regular than the body of compressor data, possibly because of the enhanced effect of, usually, undefined boundary layers in diffusing flow in the latter. Comment is made in Part 2 on the predictive and design models reported in the liturature

Key words: *turbines, compressors, turbine blades, tip gap*

The need for a knowledge of the flow and hence the losses in the blade tip regions of turbomachines is highlighted by the requirement for high compressor pressure ratios leading to small annulus heights and relatively large tip gaps in a new generation of high thermal efficiency gas turbine engines. Only in quantifying the tip flows in suitable models will design procedures be established to account fully for tip loss effects, permitting design to minimise these effects.

Part 1† of this review[1] examined the experimental research into the aerodynamics of turbomachinery tip flows using cascaded aerofoils. That work[2–18] yielded a body of detailed observations which gave a good physical understanding of cascade flows with blade tip gaps varying from zero to relatively large values. While some experimental facilities[10,12] incorporated devices to simulate rotor/casing relative movement, the effects of rotation could only be modelled in an incomplete manner. Flow models whose proposition was based upon cascade results (see for example Refs 7, 8 and 10) were seen neither to account fully for rotational effects, nor did they allow correctly for the tip vortex distribution in the region of the tip gap. Cascade results do not then preclude the need to pursue examination in the experimentally more difficult environment of real turbomachines, but rather underline this need while providing some evidence of the physics prevailing in tip gap regions.

Part 2 of this review considers the tip gap research executed in rotating machines, both compressive and expansive.

Experiments in compressors

Because of the historical lack of suitable high response rate instrumentation and data logging facilities, most measurements on rotating machinery have not been made to the same detailed standard as those on cascades. In general, the effect of tip gap variation on overall performance parameters, work coefficient and efficiency has been examined. In experiments using water as a working medium, such measurements have been supplemented by visual observations and, in certain experiments, radial traverses have been made. Currently proposed research programmes, however, are capable of exploiting modern instrumentation for detailed flow evaluation.

Effect of tip gap on overall performance parameters

In one of the earliest experiments in which the tip gap was varied in a controlled manner, Ruden[19] used a single stage fan. His data established the shape of that to follow, an almost linear relationship showing both efficiency and work coefficient to reduce with increased tip gap. Six tip gaps were employed from

† Nomenclature is defined in Part 1 of this review[1] where Figs 1–8 and Refs 2–18 also appear.

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$0.002 < \lambda/D < 0.012$† and the operational characteristics in terms of work coefficient and efficiency were progressively depressed with increase in tip gap (Fig 9). The observed linear relationship between efficiency and work coefficient with tip gap respectively resulted from cross plotting these data on lines of constant mass flow function (Fig 10).

Quoting results from an industrial compressor, De Haller[20] recorded an almost linear drop in peak efficiency from 93% to 84% as the tip gap was quadrupled and Fickert[21] found a 3% fall in efficiency of a machine when the radial clearance was doubled. Much of this earlier work covered a range of tip gap size larger than that normally employed in current designs, but the data did serve to establish trends which have continued to be confirmed, with one or two notable exceptions. The rate of efficiency loss was generally from 1.25–1.50% per 0.25 mm (0.010 in) increase in gap size.

Rains[22] investigated a three stage axial water pump at design point operation. Tip clearance, which was controlled by shims fitted beneath the rotor blade root, was altered from 0.203–0.906 mm (0.007–0.035 in) a dimensionless range of 24% of the blade tip section maximum thickness or 1.4% of the tip chord. Efficiency fell by 3.5% (Fig 11) and the work coefficient, defined as the work input over the dynamic pressure associated with the tip speed, fell by 0.007 (Fig 12), a slightly higher rate than that noted by Ruden who measured the same loss over a larger range of tip gap.

Rains' work, however, was aimed mainly at investigating cavitation. Since cavitation was formed where the fluid pressure dropped to the vapour pressure and the region of lowest pressure was the core of the tip generated trailing vortex, it had been established[23] that the inception point of cavitation, and hence the point of lowest static pressure, was in the tip region and not on the blade surface as traditionally supposed. Rains' observations have value in the gas compressor field. He noted that for very large tip clearances the geometry of the vortex was similar both for rotor and stator. With small tip clearances for a stationary blade the origin of the vortex was just behind the quarter-chord position, an observation that was closely aligned with that of Lakshminarayana and Horlock[7] who used this as geometric input for a subsequent model. With rotating blades, however, Rains observed that the vortex was locked to the leading edge of the blade profile. Defining a parameter k, the cavitation number, which can be understood to increase as the cavitation performance decreases, he noted that, instead of a linear

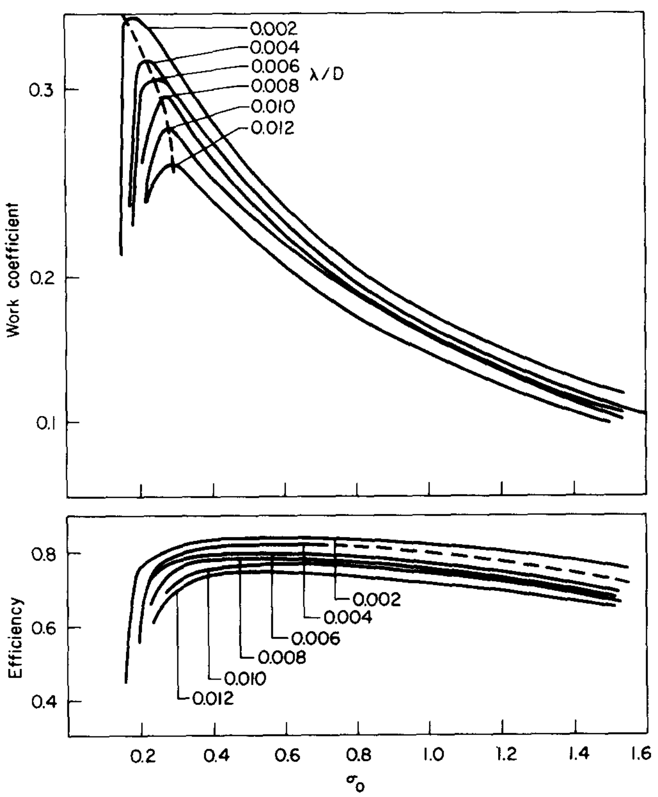

Fig 9 Experimental characteristics for various clearances between rotor blades and housing[19]

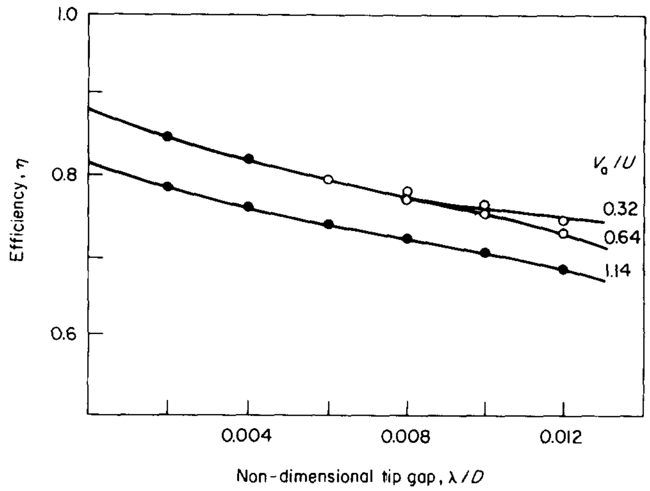

Fig 10 Effect of tip gap on efficiency[19]

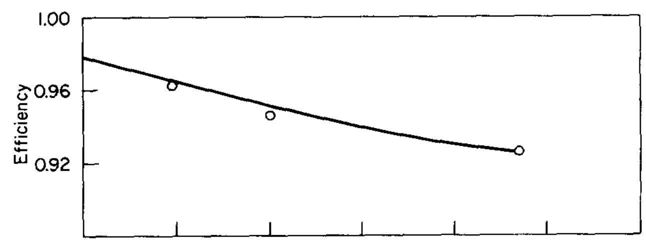

Fig 11 Variation of the efficiency with rotor tip clearance[22]

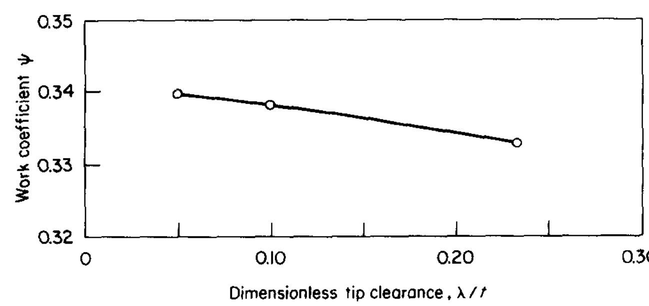

Fig 12 Variation of the work coefficient with tip clearance[22]

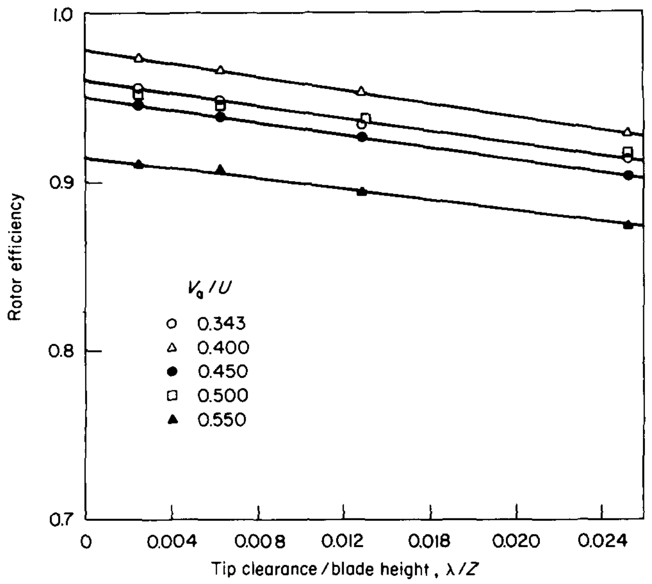

Fig 13 Rotor efficiency as a function of tip clearance for various flow rates[24]

variation of k with tip clearance, there was a region of small tip clearance which showed a marked decrease in performance, possibly the first indication of non-linearities resulting at small tip clearance. Rains also noted that the tip clearance flow was appreciably increased as a result of the rotor/wall relative velocity, confirming the cascade work of Gearhart[12] and Dean[10].

Rains' investigations were extended by Williams[24] to off-design operation of the same pump, but in single stage configuration. While most of his work was at constant rotational speed, the mass flow was varied and, while it approached stall, did not enter it. The range of tip clearance was selected to bracket that used in turbomachinery practice, $0.00368 < \lambda/c < 0.03742$. There was some scatter in Williams' results, but he found that the pressure coefficient and the efficiency decreased approximately linearly with gap size, the maximum rate of change being at the lowest flow rate used corresponding to the highest pressure coefficient (Fig 13). Cross plotted in terms of efficiency against flow coefficient, a series of well recognised curves with a maximum efficiency at the design mass flow resulted, the curves being depressed progressively with increased tip clearance, a confirmation of the trends observed first by Ruden[19].

In addition to their own results (discussed later), Jefferson and Turner[25] also re-presented some earlier results of Bowden and Jefferson[26] from an eight stage axial compressor. Over the range of tip gap which varied from 0.264–1.372 mm (0.009–0.054 in) $(0.0144 < \lambda/c < 0.0864)$ there was an almost linear drop in peak efficiency and a fall in temperature rise coefficient (Fig 14) whose rate increased progressively with tip clearance. Although their commentary does not mention it, the data suggest an initial improvement in stability limit line with subsequent slight decay.

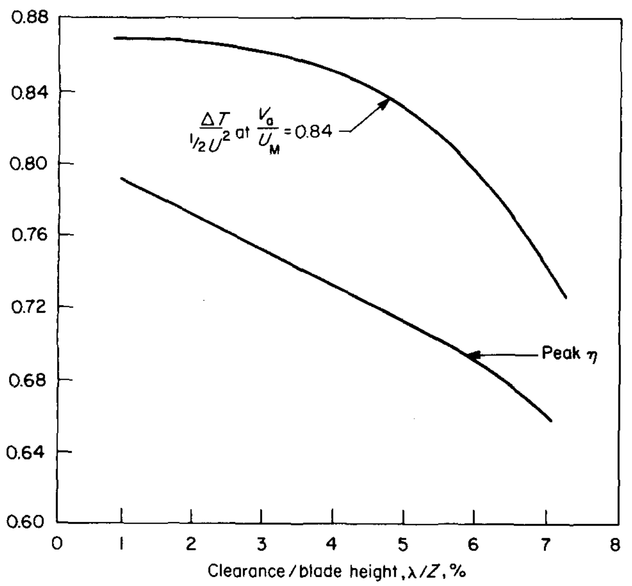

Fig 14 Eight stage compressor. Variation of overall parameters with percentage tip clearance[25,26]

Radial survey data

Williams[24] conducted surveys of axial velocity behind the rotor row. These indicated a velocity decrement towards the tip region in keeping with the inlet boundary layer profile. At flow rates above the design value there was little change of velocity with variation in tip gap, but at flow rates less than the design value the effect of the largest tip gap was marked over the outer 25% of the blade height. It was at high mass flows, however, that the effect of high tip clearance reduced locally the work coefficient. These data are commensurate with reduced turning of the flow because of tip leakage, the effect covering the outer 25% of the blade height. Early work on the subject of tip clearance effects was reported by Hutton[27]. Using a very low solidity fan he conducted tests at tip gaps of $\lambda/Z = 0.5$, 1.5, 2.5, 3.5 and 4.5%. He noted that alteration of the tip gap led to substantial changes in the exit axial velocity (Fig 15) and swirl angle over the whole of the

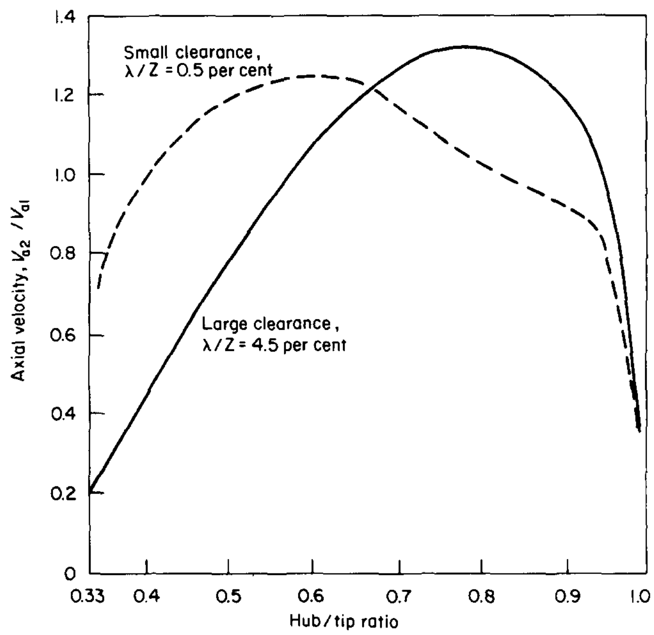

Fig 15 Effect of tip clearance on the distribution of axial velocity[27]

blade height. The effect of tip gap was thus more global in its extent than that observed by Williams. In comparing the data from the two extreme gaps used, Hutton found that the effect on static pressure rise extended over all of the blade height leading to an almost linear decay of work coefficient with tip gap (Fig 16).

Stability limit considerations

One important observation made by Hutton[27] and yet strangely not apparently pursued by subsequent workers, was the evident change in the compressor stability limit operation with tip gap (Fig 17), Certain other work has identified the problem of surge margin erosion with tip gap size. Hutton found though that, in addition to the stability limit being encountered at higher mass flow coefficients with increase of tip gap, the strong hysteresis effect was removed. Recent experiments of Peacock and Das[28] supporting an examination made by Greitzer[29] confirm that the jumps associated with the hysteresis indicate changes in the rotating stall pattern. Hutton's measurements then indicate that the pattern of rotating stall may be governed by the tip gap geometry, a point not considered thus far either in rotating stall investigations or tip gap experiments.

Jefferson and Turner[25] pursued tests on an air compressor of six axial stages and low overall pressure ratio, using a range of blade designs and tip geometries. Most of their work was conducted with shrouds on the rotor and/or stator rows and the effects of varying shroud clearance were examined. In this case, clearance was defined as the axial distance of the shroud knife-edge and the compressor sealing face, so the geometry employed could not be equated with that under review. The conclusions nevertheless have qualitative value in that with increased clearance, yielding higher tip leakage, the efficiency at any particular mass flow coefficient fell giving a lower pressure rise coefficient. The stability limit of the compressor, however, was delayed to a lower flow coefficient, an observation that was in direct contradiction to that of Hutton[27]. In comparative tests using consecutively shrouded blades and unshrouded blades with tip clearance they found

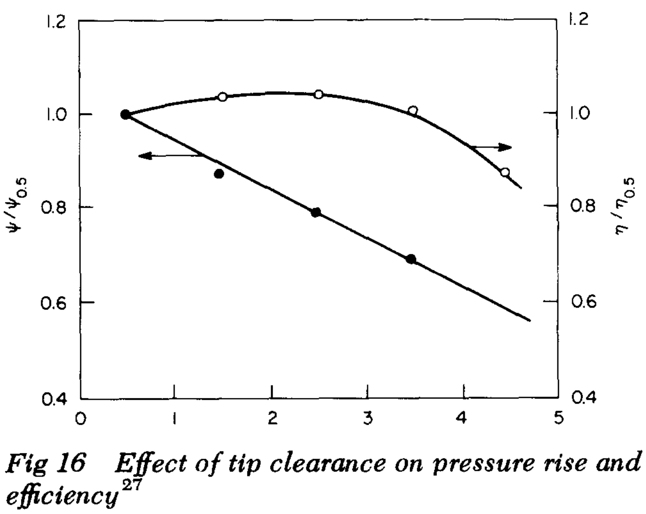

Fig 16 Effect of tip clearance on pressure rise and efficiency[27]

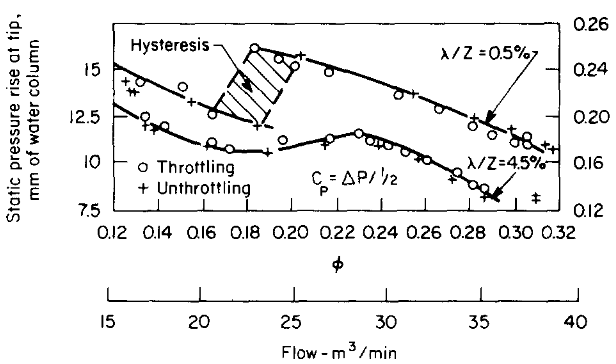

Fig 17 Effect of tip clearance on stalling and wall pressure rise[27]

that better performance resulted in having a tip clearance geometry and this was associated with suppression of a blade stall condition with the presence of the tip gap.

Tip gap optimisation

Spencer[30] used a four bladed axial flow water pump for his investigations which included tip clearance variation from 0.6–2.4% of the blade height. He found that the effects of tip clearance was the greatest at low flow rates when the pressure differences at the gap were at their greatest. The 1.8% increase in clearance (based on blade height) yielded a 15% drop in design flow and an overall drop in efficiency. There was an initial improvement in efficiency as the tip gap was opened (Fig 18) and though, in his paper, Spencer thought the effect to be spurious, there were in the associated Communications observations by Desmur[31] who cited the work of Ténot[32] and Daily[33], by Medici[34] and by Yamazaki[35] that there was an optimum tip gap for operation. Such observations were in line with that of Rains[22]. Although all of their data were not consistent at small tip geometries, Jefferson and Turner[25] concluded that a small tip gap, of the order of 1% of the blade height, would be beneficial in a design, a conclusion in sympathy with the communications[31,34,35] relating to Refs 27 and 30. This admitted the possibility of an optimum tip gap, indicated by Lakshminarayana and Horlock[7,8] although in general not confirmed by the later main corpus of data available from the range of workers in the field.

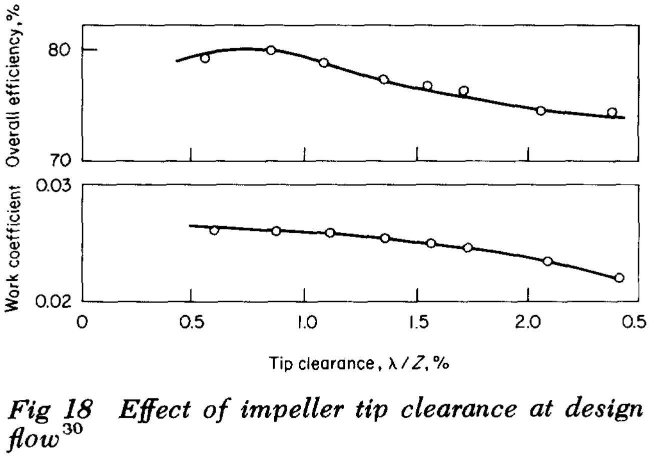

Fig 18 Effect of impeller tip clearance at design flow[30]

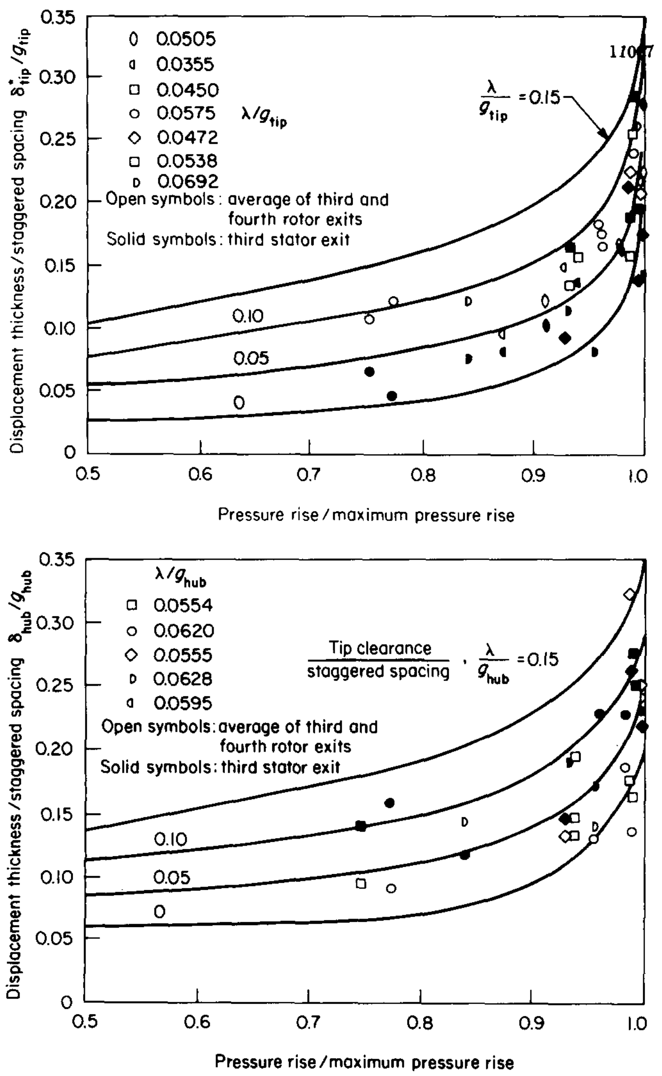

Fig 19 Displacement thickness of casing boundary layers[38]

The concept of an optimum tip gap was further pursued by Lakshminarayana[36] who postulated that, since the secondary distributed circulation and the tip leakage flow were mutual opposition at the tip, an optimum should exist when the sum of the two associated secondary circulations was zero. He found a qualitative agreement in applying this principle to the results of Dean[10] and Hubert[37].

Annulus boundary layer considerations and peak pressure rise

Based upon earlier results from a 12 stage compressor Smith[38] concluded that, by the fourth stage, essentially repeating conditions in terms of velocity profile had been reached. Using, in consequence, a four stage low speed compressor as the vehicle for his investigation and considering data from the third rotor, third stator and fourth rotor only, he conducted a comprehensive series of tests with a large range of geometric variables. Arguing that the staggered pitch was a better reference dimension than any other for non-dimensionalising parameters, he produced curves of constant non-dimensional tip clearance on a non-dimensional displacement thickness versus non-dimensional pressure rise field, both for the casing and hub regions of compressors (Fig 19). The data indicated the powerful effect of the magnitude of the compressor pressure rise upon the annulus displacement thickness for any tip gap. It also showed that the displacement thickness was strongly affected by the gap size, reduced gap size permitting more energisation of the wall boundary-layer to keep it thin. A further important conclusion of Smith's work was that stall was likely to occur at a limiting value of the boundary layer displacement thickness that depended upon tip clearance, indicating that the tip clearance contributed towards control of the limiting stall mechanism. In passing, it is noted that the tip gap alone was not enough to explain the variations of stalling pressure coefficient, but that the axial gap between blade rows also contributed, the lower the gap, the higher the stalling pressure rise at constant tip clearance. Because the data to hand did not cover a wide enough range of tip gap, the curves of constant tip clearance came 'from engineering judgment seasoned by (appropriate) boundary layer data.' Plotting data for three different blade geometries in terms of the peak static pressure coefficient against the tip clearance, Smith produced a series of straight lines indicating reducing performance with increasing tip gap (Fig 20), thus adding a further confirmation to the work of Ruden, and others already cited.

Extending Smith's work and pursuing an interest in the stall pressure rise capabilities of compressors, Koch[39] identified tip gap as one of the factors influencing the stall pressure rise capability of multi-stage compressors. Varying tip clearances from 0.7% to 3.4% of the blade height by cutting the blades accordingly and using data at a non-dimensional gap of 0.055 (the normalising dimension being the average pitch-line gap) he showed that the non-dimensional static pressure rise coefficient reduced with increasing gap size and was insensitive to aspect ratio in the range 2.0 < A.R. < 5.0. While the trend was similar to that encountered by other workers, the noteworthy features are that, over the normal range of tip gap, the results were not quite linear and that at small to zero tip clearance

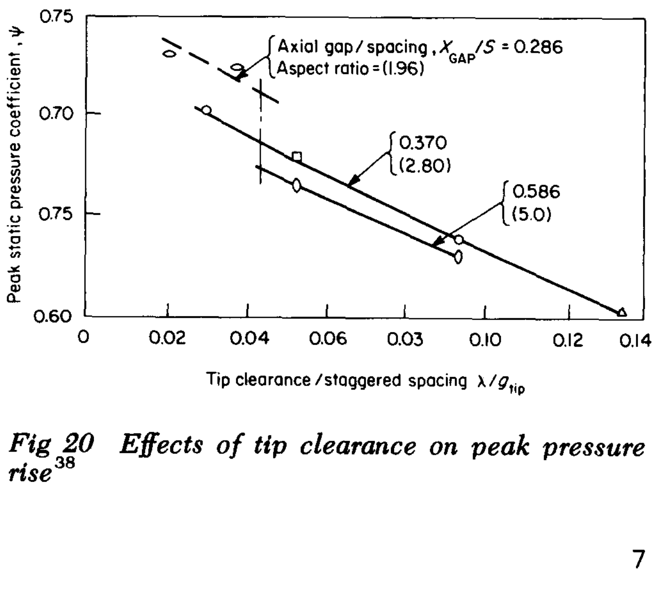

Fig 20 Effects of tip clearance on peak pressure rise[38]

the pressure rise characteristic rose sharply (Fig 21). It certainly contained no indication of an optimum tip gap.

To relate Koch's data to those of other workers it is necessary to understand the parameters in which he worked. Koch defined his stalling static pressure rise C_h as:

$$C_h = \frac{J C_p t_1[(p_2/p_1)^{\gamma-1/\gamma}-1]_{\text{Stage}} - \frac{1}{2g_0}(U_2^2 - U_1^2)_{\text{Rotor}}}{\frac{1}{2g_0}(V_{1\,\text{Rel Rotor}}^2 + V_{1\,\text{Rel Stator}}^2)}$$

Although nominated as a pressure rise coefficient, this is really an enthalpy rise coefficient which, assuming that repeating stage conditions are fairly closely obeyed (see Smith[38]) so that the stagnation pressure ratio varies as the static pressure ratio, may conveniently be written:

$$C_h = \frac{2gJC_p t_1 \eta_{AD}\left(\frac{t_2}{t_1}-1\right)_{\text{Stage}} - (U_2^2 - U_1^2)_{\text{Rotor}}}{V_{1\,\text{Rel Rotor}}^2 + V_{1\,\text{Rel Stator}}^2}$$

Other work already reviewed has indicated a linear degradation of efficiency with tip gap. Jefferson and Turner[25] and Rains[22] measured an almost linear degradation of work coefficient $\Delta T/\frac{1}{2}U_m^2$ with gap size. Since:

$$\frac{\Delta T}{\frac{1}{2}U_m^2} = \frac{(T_2-T_1)}{\frac{1}{2}U_m^2} = \frac{T_1\left(\frac{T_2}{T_1}-1\right)}{\frac{1}{2}U_m^2} \simeq \frac{t_1\left(\frac{t_2}{t_1}-1\right)}{\frac{1}{2}U_m^2}$$

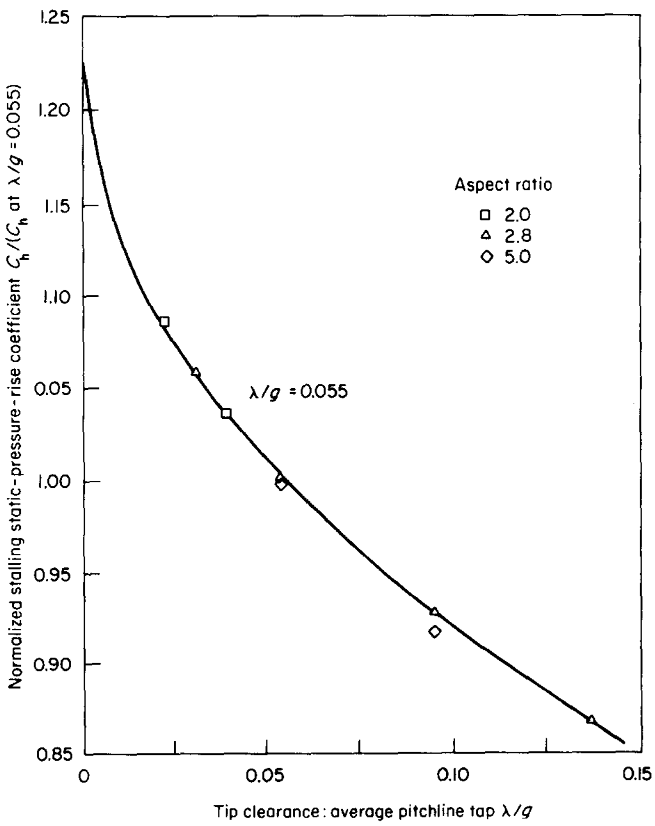

Fig 21 Effect of tip clearance on normalised static pressure rise coefficient[39]

for constant inlet conditions and blade speed, $(t_2/t_1)-1)$, it may be concluded, varies linearly with gap. We see therefore that

$$C_h = \frac{K_1\eta(\lambda)\left(\frac{t_2}{t_1}-1\right)(\lambda) - K_2}{K_3}$$

where $K_{1,2,3}$ are constants, so:

$$C_h = \frac{K_1 f(\lambda)^2 - K_2}{K_3}$$

which is quadratic in form. The apparently atypical results observed in Koch's data can therefore be reconciled with the linear trends elsewhere. It is noted, however, that at gap levels less than 0.24, Koch's curve is extrapolated.

Experiments with machines of significant pressure ratio

Most of the work so far reviewed has been executed either on essentially incompressible flow or moderately loaded machinery. Investigations carried out by Moore and Osborne[40], however, were on tip clearance effects on a transonic four stage rig. As part of a broader investigation that examined casing treatment, experiments without casing treatment indicated a decrease of pressure ratio and efficiency with increasing tip clearance and also an effect upon the stability limit of the compressor. Fan tip clearances from 0.61 cm to 1.78 cm were employed and the fan, whose design overall pressure ratio was 1.75 was tested from 50% to 100% of its design speed. The trend of efficiency and pressure ratio change with tip gap size, while similar to that of other data was not quite linear (Fig 22), the rate of loss being greater with tip gap change at small tip gaps. In a transonic machine, the profile loss always rises sharply in the outer region of the blade because of the presence of shock waves and shock/boundary layer interactions. From Ref 41 it is seen that a substantial part of the rotor blade had a supersonic entry Mach number. The non-linear form of Moore and Osborne's data may then be due to the progressive removal of the radially non-linear loss associated with the shock system as well as the effects of the annulus/gap interaction.

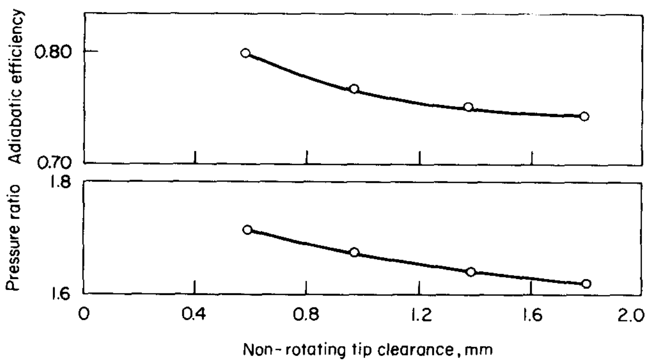

Fig 22 Effect of tip gap on efficiency and pressure ratio[40]

The results of Lindsey[42] should also be cited here. His work was in the development of the Armstrong-Siddeley Mamba engine where, although stage pressure ratios were significant, yielding an overall pressure ratio on three stages tested of about 1.7:1, there was no significant supersonic flow. Tests indicated an almost linear drop in efficiency with increased tip clearance (Fig. 23). It is mentioned, however, that there was considerable scatter in Lindsey's data.

Experiments in turbines

The centre of interest in tip gap effects in turbines is with the small diameter machine in which the tip gap is always comparatively large related to any relevant machine dimension. Its effect upon a small machine is in consequence larger.

As a means of extending observations on the water-table to a rotating machine, Booth, Dodge and Hepworth[15] used a low aspect ratio turbine in which the tip gap was varied from 1% to 3% of the blade height. The design point total-to-total efficiency fell from 92% to 89%; 1.5% for every 1% of blade height.

Ewen, Huber and Mitchell[43] considered the blade tip clearance as one feature affecting the aerodynamic efficiency of a turbine. Progressively altering the outer diameter of the annulus, they found that in the range $0.178\,\text{mm} < \lambda < 0.711\,\text{mm}$ the efficiency fell progressively, but in a non-linear manner, with increase in tip clearance (Fig 24). The nature of the non-linearity, a greater loss for increments in gap dimension at small gap, was qualitatively similar to the fan data of Moore and Osborne[40]. In this, the data of Ewen, Huber and Mitchell appear to be unusual, for their turbine was subsonic and most other turbine data do not show the observed effect at small gap heights.

Early work on a small impulse turbine by Kofskey[44] yielded a linear 1.75% reduction in turbine efficiency for a 1% increase in annulus height. Extending the investigation to a two-stage turbine with high reaction in both stages and using two tip clearances corresponding to 1.06% and 2.47% of the average annulus height, Kofskey and Nusbaum[45] also detected a linear relationship in efficiency degradation. The rate of efficiency shedding was higher, however, and this was attributed to the higher reaction, which would yield a larger pressure difference across the blade tip overall.

These trends were confirmed by Futral and Holeski[46,47] on a single stage high reaction turbine. A 20% change in static efficiency (0.79 to 0.63) was measured in increasing the rotor tip clearance from 1.2% to 8% of the blade height and the slope of the straight line was closely similar to that obtained by Kofskey and Nusbaum[45].

Szanca, Behning and Schum[48] used a 10.0 in (250 mm) diameter turbine of 80.5% tip reaction to establish a closely linear relationship between the efficiency decrement and the rotor tip clearance. This was varied from 2.3% to 6.7% of the annulus height by progressively machining the rotor blade tips.

Haas and Kofskey[49,50] investigated the effect of the rotor tip on a 5.0 in (125 mm) turbine, but varied the geometry to include both a recessed wall with extended rotor blade as well as a plain outer wall and normal tip geometry. While the rate at which efficiency loss with increase in tip gap was lower with the recessed wall geometry, in both cases, straight line relationships resulted. Since, in the case of the recessed wall geometry, the blade tip gap was always within the recess and therefore not subject to free-stream or even normal wall boundary-layer conditions, but may have been in a quasi-stationary region within a flow separation created by the recess, so it may be anticipated that the tip gap effect was somewhat suppressed.

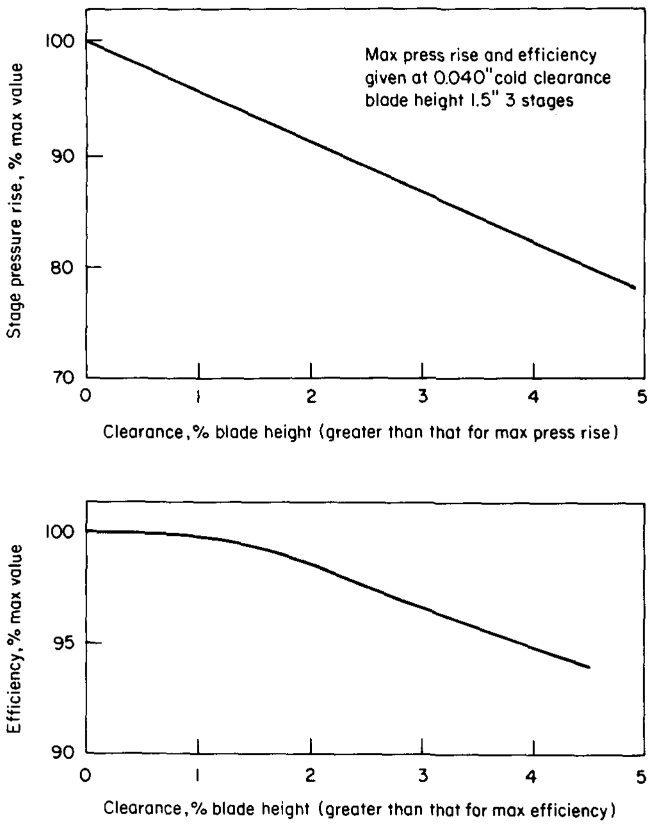

Fig 23 Effect of tip clearance on compressor performance[42]

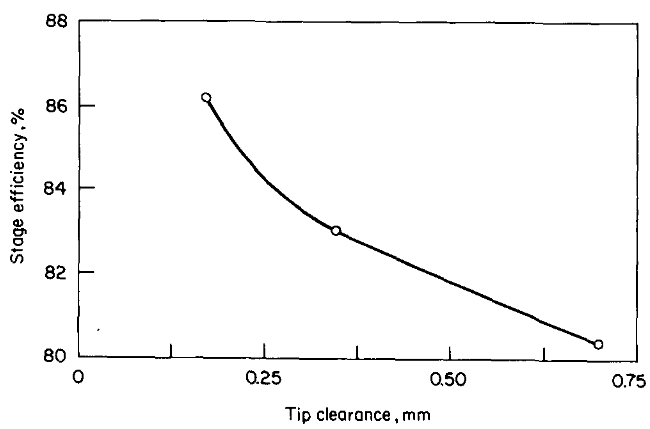

Fig 24 Effect of blade tip clearance on turbine stage efficiency[43]

Haas and Kofskey's papers[49,50] are particularly valuable in that they produce a synoptic graph showing data from a range of workers (Fig. 25). From this, a general conclusion can be drawn, that the greater the reaction, leading to increased pressure difference at the blade tip, the greater the penalty to efficiency of increased tip clearance.

Tip gap effect on mass flow

A turbine rotor represents a resistance to the flow of a working fluid in an annulus and it may be anticipated that increasing the tip gap would reduce this resistance, permitting under similar inlet conditions an increased mass flow.

This was measured by various workers and, in particular, Kofskey and Nusbaum[45] and Holeski and Futral[47] (Fig 26) found that the relationship was linear. Such a relationship might be anticipated since the variation of the tip gap area is linear with tip gap height as long as the tip gap is small compared with the annulus outer diameter.

Since the increased mass flow is likely to be passing through the gap region rather than the turbine rotor channel, this would not affect the turbine work coefficient which would change directly with the measured efficiency.

Spanwise effects of tip clearance

Detailed measurements by a number of workers have indicated that the effects of changing the tip gap, while profound in the tip region, are not confined to it. Ewen, Huber and Mitchell[43] measuring the efficiency at different radial stations, found that, in the tip region, up to 15% loss of efficiency could be measured in quadrupling the tip gap (Fig. 27). While it reduced towards the hub, the decrement was still measurable at 40% of the blade height. Kofsky and Nusbaum[46] measured the effect upon the exit stagnation pressure of increasing the gap 2.40 times to extend about 14% of the blade height from the tip and over this region the exit angle also increased, thus contributing to the loss of efficiency. Holeski and Futral[47] and Szanca, Behning and Schum[48], however, measured an increased flow angle with tip gap over the whole of the blade height (Fig 28(a)), while Holeski and Futral's data indicated that the change in stagnation pressure remained fairly localised in the outer 20% of the blade height (Fig 28(b)). Contrary to the data of Ewen, Huber and Mitchell[43], Holeski and Futral also measured an efficiency decrement over the whole blade height (Fig. 28(c)).

It is clear from these data that the effect of the tip gap aerodynamics upon the aerodynamics of the whole blade is very large and it may be reasoned that design procedures used over the whole blade height should include an allowance for the tip flow.

Observations on compressor experiments

The data which have been published on tip gap effects measured experimentally on compressors

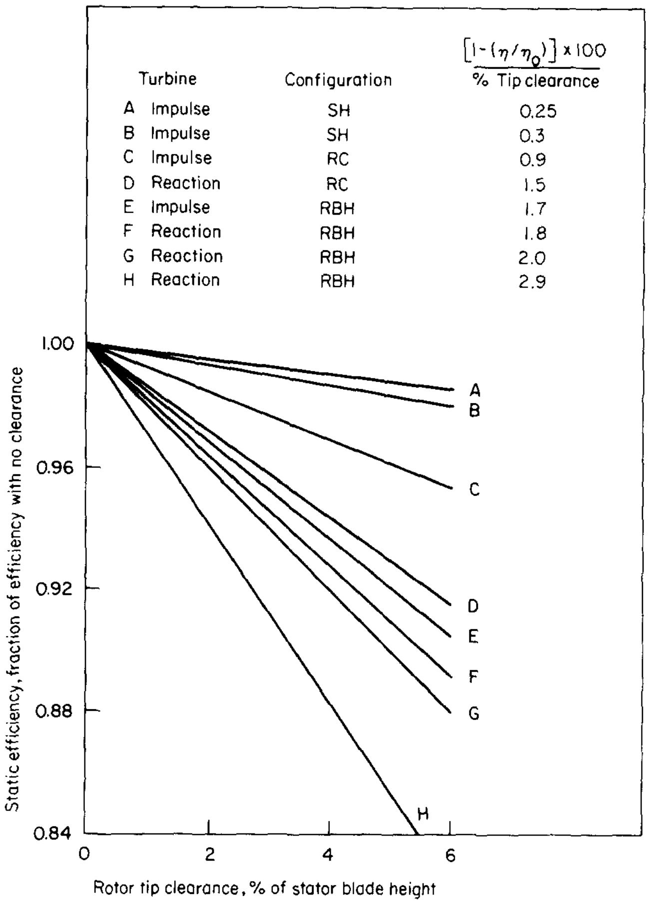

Fig 25 Effect of rotor tip clearance on performance for various turbines[50]

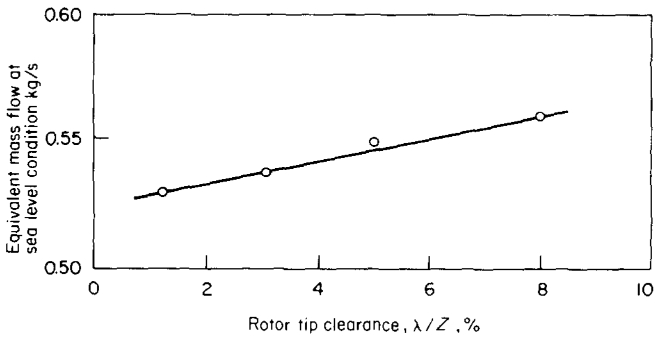

Fig 26 Effect of rotor tip clearance on equivalent mass flow at design equivalent speed and pressure ratio[47]

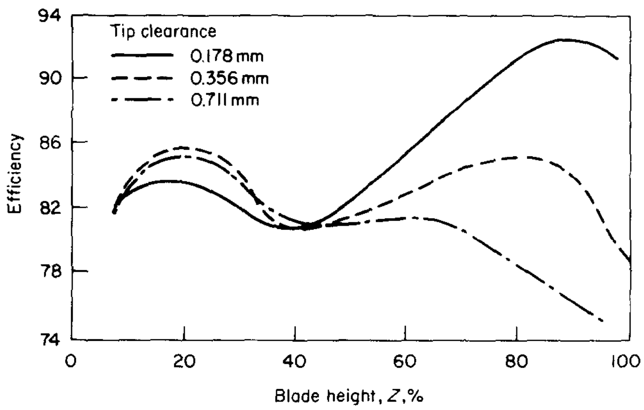

Fig 27 Effect of blade tip clearance on spanwise efficiency distribution[43]

covers nearly fifty years and, considering the duration of investigation, is rather sparse. In view of the developments in instrumentation and techniques over that period and of the profound improvements in turbomachines, the general consistency of the data are nevertheless the more remarkable.

Few measurements exist of the radial variations of the flow within the annulus, but of those that do, Hutton[27] shows an effect of tip clearance

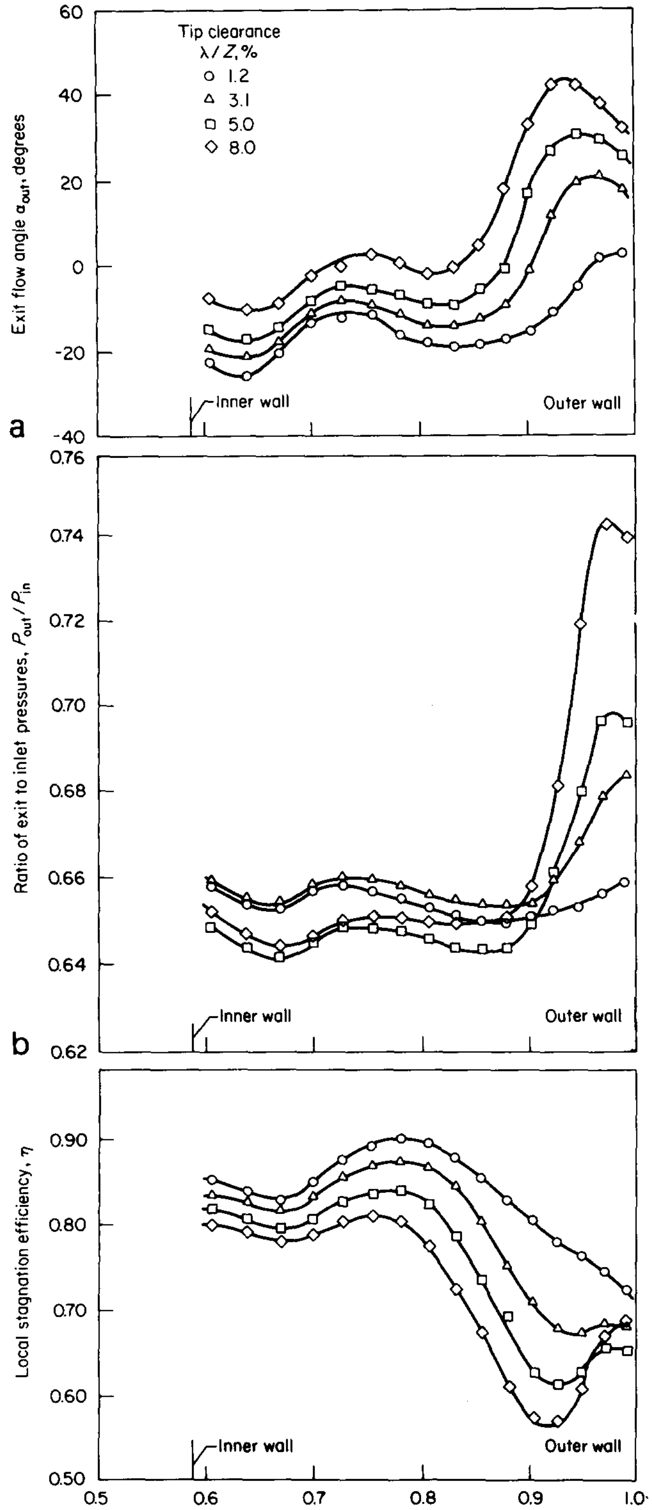

Fig 28 Survey results at rotor exit at design equivalent speed and pressure ratio[47]

that dominates the tip region aerodynamics and has a marked effect over most of the annulus height. This indicates that beyond installing a simple loss statement in the calculations for the tip region or even allowing for recambering of the blade in the wall boundary-layer region as suggested by Daly[33] in designing compressors to allow for tip gap effects, consideration should be given to the effect over the whole of the flow field. Williams[24] measured data which, while showing a marked effect of tip clearance, found that it was mainly confined to the outer 25% of the blade height.

The most widely available data consists of the measurement of loss in overall pressure rise, work coefficient and efficiency with increase in tip gap. Since with increase in tip gap the reduction in swept area of the blade is $d\lambda \pi D$, linear with $d\lambda$, the change in tip gap and the work derives from the change in whirl velocity across a rotor row, a linear fall in work coefficient is reasonable. A linear change in work coefficient with an associated linear change in efficiency would not produce a linear change in pressure rise as has been indicated in assessing Koch's data[39]. Bowden and Jefferson[26], Lindsey[42], Hutton[27], Moore and Osborne[40] and Smith[38] all indicate a linear response of pressure coefficient, while Jefferson and Turner[25], Ruden[19] and Rains[22] discovered a closely linear response of work coefficient to tip gap. It therefore remains to be seen which, if either of these parameters is, in general, linear in response.

A further point of disagreement between the data examined concerns efficiency variation. The measurement of efficiency always carries problems, but while absolute levels may be difficult to guarantee, relative changes on one machine in which the tip gap is being changed should maintain good internal consistency. The general linearity of response of efficiency with tip gap is confirmed by most researchers, Hutton[27] being the exception. At very small tip gaps, however, a peak in efficiency was noted by Spencer[30] and confirmed by others and a similar trend was noted by Lindsey[42] suggesting, as Lakshminarayana and Horlock[7,8], that an optimum gap exists. The work of Ruden[19], Rains[22], Williams[24], Bowden and Jefferson[26] and Koch[39] indicates no optimum, a progressive rise in efficiency being measured with reducing tip clearance. Whether an optimum gap exists in a reasonable engineering range of size remains to be seen.

Stability limit is always of importance to the turbomachinist and the data indicates that this can be affected by the tip clearance. Most workers who measured the stability boundary of their machines found that an increase in tip clearance caused the stability limit to be encountered at higher mass flow and lower pressure rise. These included Ruden[19], Hutton[27], Smith[38], and Moore and Osborne[40], whose data indicated that the greatest change took place while the gap was still small. Bowden and Jefferson[26] produced a result not repeated elsewhere, indicating an initial reduction in the stall mass flow before a later modest increase with tip gap size. Spencer[30] using a water pump indicated that the cavitation line was moved to lower mass flows with increasing tip gap, suggesting that the very low pressures associated with tight vortices shed at the tip were relieved. Jefferson and Turner[25] using various shrouded geometries showed that with increasing shroud gap the stall line was forced consistently to lower mass flows, but that an increased stator gap (unshrouded) moved the line to higher mass flows.

While the general consensus is that an increased tip gap affects the stability limit margin deleteriously, Jefferson and Turner's data, which must be regarded as atypical because of the quite different geometry, nevertheless indicates that the effect is not a consequence of mere leakage flow past the tip, but must be because of complex aerodynamic interactions as yet to be understood.

Within the research area of unsteady flows in turbomachines, rotating stall has long been of interest, but little is yet known of the detailed mechanics of the stall cell. It is reasonable to assume that for a tip located stall cell, the mechanics may be affected by tip gap flows and it is significant that Hutton[27] found, not only a change in the position of the stability limit line but also in its characteristic, a strong indication of a change in stall geometry and mechanics.

Predictive methods for use in compressor design and analysis

Of the various approaches to predicting the loss in compressor efficiency due to the presence of the tip gap, that due to Lakshminarayana[36] is based upon earlier cascade measurements[7,8] and that due to Smith[38] is based upon measurements made with a multi-stage low speed compressor.

Lakshminarayana[36] derived a semi-theoretical expression for predicting the decrease in efficiency due to clearance. His model was based upon use of the fraction of aerofoil lift retained at the blade tip which he approximated by the equation:

$$(1-K)=0.23+7.45(\lambda/S)$$

in the range $0.01<\lambda/S<0.10$. Using an expression for the induced drag in inviscid flow which included the retained lift term, the lift coefficient and geometric details both of the blade and the gap, he derived an expression for stagnation pressure loss due to the potential vortex. To this he added a term to account for the kinetic energy associated with the spanwise flow in the blade boundary layer. Dividing the sum of the total pressure loss by the isentropic pressure rise in the machine he found that:

$$\Delta\eta=\frac{2\Delta P}{\rho U^2\psi}$$

Making simplifying assumptions for the displacement thickness of the boundary layers on both suction and pressure surface there resulted the semi-empirical relationship:

$$\Delta\eta=\frac{0.7\lambda\psi}{Z\cos\alpha_m}\left[1+10\left(\frac{\phi}{\psi}\cdot\frac{\lambda A}{Z\cos\alpha_m}\right)^{1/2}\right]$$

Lakshminarayana found good agreement in predicting the results of Jefferson and Turner[25] derived from Bowden and Jefferson[26], and with the data of Williams[24], Ruden[19], and Spencer[30]. The model diverged somewhat from the data of Koleshnikov[51].

In an attempt to predict the flow field in the tip region of a blade, Lakshminarayana introduced a theoretical model which took into account the presence of a vortex core. Within a core radius predicted by Rains[22], Lakshminarayana assumed forced vortex flow and outside this radius, free vortex flow. The location was determined by knowing the flow field due to image vortices, as in his experiment, and assuming that the vortex originated at the leading edge, an observation of Rains[22]. Extending Lamb's solution[52] for the induced flow field of an infinite row of point vortices to two infinite rows, it became possible to predict the velocity components in the field and the air angles. Lakshminarayana acknowledged that the model based upon inviscid assumptions was unable to predict the flow losses and that, according to Newman[53], the major departure between an inviscid and viscous model, so far as rotational velocities were concerned, was confined to the region at the outer edge of the core. Introducing Newman's equation[53] and modifying it by use of Batchelor's solution[54] for the deficit in longitudinal velocity, Lakshminarayana produced an average stagnation pressure loss equation of the form:

$$\frac{P_1-P}{\frac{1}{2}\rho V_1^2}=\int_0^1\frac{(P_1-P)_{y/s}}{\frac{1}{2}\rho V_1^2}\,d\left(\frac{y}{s}\right)$$

The model predicted the flow picture from earlier cascade results[8] qualitatively with good agreement in the spanwise distribution of the passage averaged air leaving angle. Using the observation of Lakshminarayana and Horlock[8] that the strength of the secondary flow was proportional to the shed vortex strength:

$$\Gamma=(1-K)\Gamma_{2D}$$

Adkins and Smith[55], assuming that the induced pitchwise-average cross-passage secondary-flow angle distribution could be obtained by taking the vortex core to have vorticity uniform in the pitchwise direction and varying as a sine-wave first half cycle in the spanwise direction, modelled the tip clearance secondary flow in a design method. They took the spanwise influence of the vorticity to be 6.5 times the tip clearance and they also allowed for some loss in the cascade pitchwise average turning by reducing the primary flow turning parameter ($\tan\alpha_1-\tan\alpha_2$) by an amount related to the shed vortex strength. This amount was determined as ($\alpha_2^*-\alpha_1$) and was gained from the equality:

$$(\tan\alpha_1-\tan\alpha_2^*)=K^*(\tan\alpha_1-\tan\alpha_2)$$

where $(1-K^*)=\frac{1}{4}(1-K)$ with the factor $\frac{1}{4}$ determined empirically.

Smith[38] used his measurements, which have already been reviewed, in a technique to predict how the pressure-flow and efficiency-flow characteristics were affected by the tip gap aerodynamics. This involved a two-step calculation using, initially, cascade data to determine performance at various spanwise stations and then superimposing the effects of the hub and tip boundary layers. To do this Smith defined the displacement thicknesses of the boundary layers in the traditional manner:

$$\delta_h^* = \frac{1}{r_h \tilde{V}_{xh}} \int_{r_h}^{r_h+\delta_h} (\tilde{V}_x - V_x) r \, dr$$

$$\delta_t^* = \frac{1}{r_t \tilde{V}_{xt}} \int_{r_t-\delta_t}^{r_t} (\tilde{V}_x - V_x) r \, dr$$

and introduced a tangential force loss in the endwall boundary layers in a similar format:

$$\nu_h = \frac{1}{r_h \tilde{F}_{yh}} \int_{r_h}^{r_h+\Delta_h} (\tilde{F}_y - F_y) r \, dr$$

$$\nu_t = \frac{1}{r_t \tilde{F}_{yt}} \int_{V_t-\Delta_t}^{V_t} (\tilde{F}_y - F_y) r \, dr$$

where ν_h and ν_t represented the amount that the tangential component of the blade force was reduced from its free-stream value by the presence of the boundary layer.

For high hub/tip ratio stages this yielded the approximate relationship for efficiency:

$$\eta = \tilde{\eta}\left[1 - \frac{\delta_h^* + \delta_t^*}{h} \Big/ 1 - \frac{\nu_h + \nu_t}{h}\right]$$

Smoothed data relating the hub and tip boundary layer displacement thicknesses δ_h and δ_t, and the hub and tip tangential force thicknesses ν_h and ν_t to the pressure rise as a fraction of its maximum value and the tip gap size were necessary. With respect to the assessment of boundary layer thickness, reasonable curves could be drawn through the data, but the data relating to the tangential force thickness had a high level of scatter, from which he chose a constant value of 0.65 of the displacement thickness as representative. Smith was able to conclude, however, that since the value of tangential force thickness was positive, stage efficiency would be automatically reduced and would, for example, be less than that predicted by Mellor and Strong[56].

The mass flow coefficient, allowing for the presence of the boundary layers was assessed in the knowledge of the displacement thicknesses of the hub and tip boundary layers, thus:

$$\phi = \tilde{\phi}\left|1 - \left(\frac{\delta_t^*}{g_t} + \frac{\delta_h^*}{g_h}\right)\overline{g/h}\right|$$

A somewhat simplified form of Smith's model was used in a method of Koch and Smith[57] to establish the design point efficiency of a multi-stage compressor. They lumped the hub and tip boundary layer data, both displacement thickness (Fig 29) and tangential force thickness (Fig 30) together,

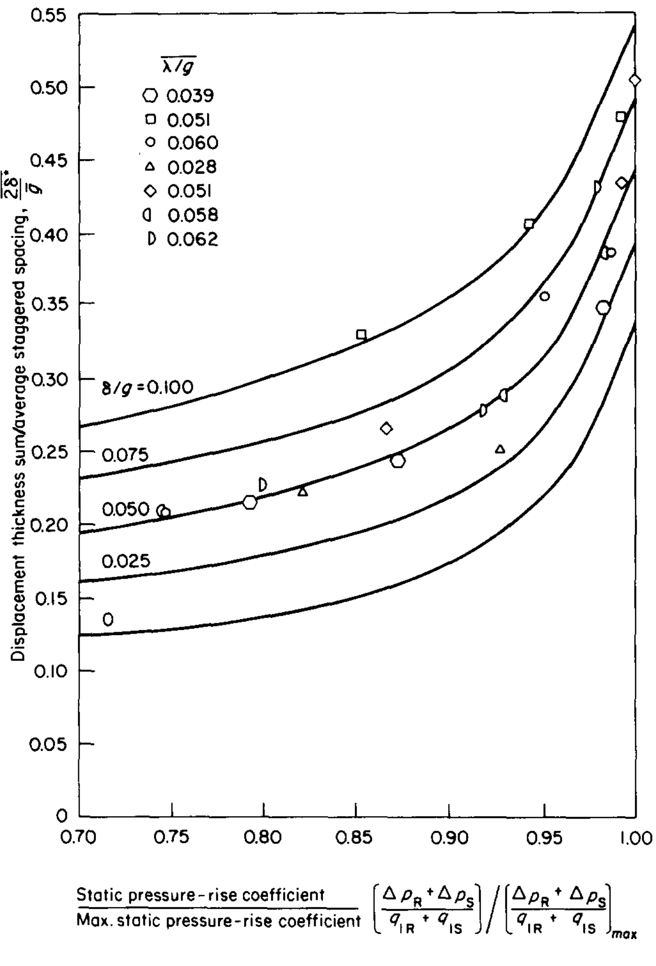

Fig 29 Sum of hub and tip end wall boundary layer axial velocity displacement thicknesses[57]

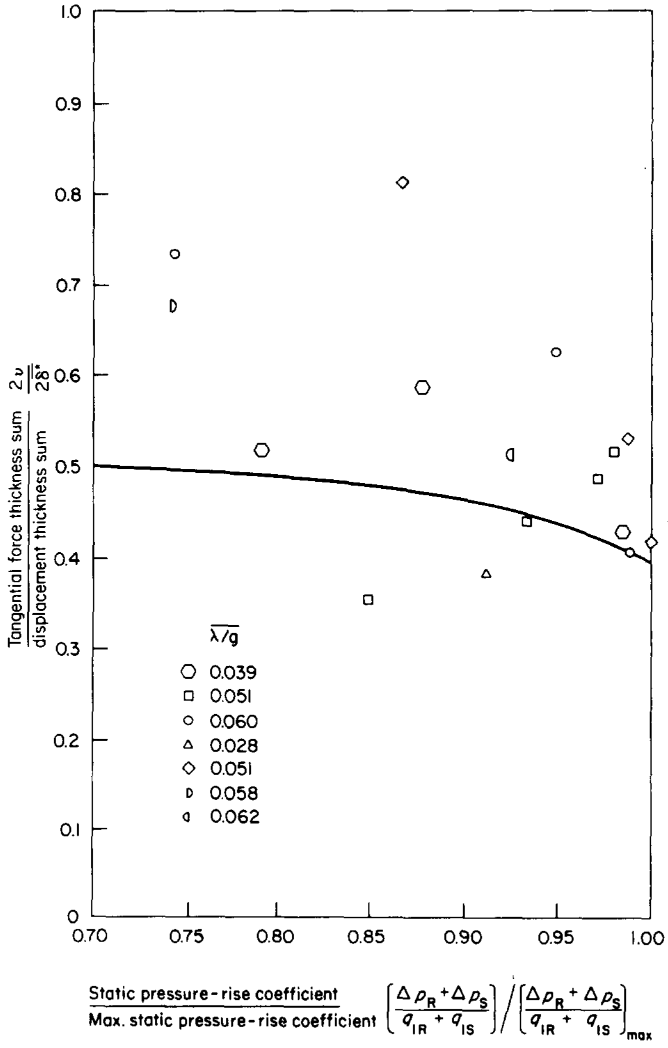

Fig 30 Sum of hub and tip end wall boundary layer tangential-force thicknesses[57]

producing a relationship between the non-dimensionalised sum of the two displacement thicknesses and the non-dimensionalised pressure rise coefficient thus:

$$\frac{2\overline{\delta^*}}{\bar{g}} = \left|\frac{2\overline{\delta^*}}{\bar{g}}\right|_{\overline{e/g}=0} + 2\frac{e}{g}\left|\left\{\frac{\Delta p_R + p_S}{q_{1_R} + q_{1_S}}\right\} \Big/ \left\{\frac{p_R + p_S}{q_{1_R} + q_{1_S}}\right\}_{\max}\right|$$

As with the individual tangential force thickness data, the combined data also contained a great deal of scatter. Through this the authors put a simple line as representative. Their method also allowed for a modification in the boundary layer displacement thickness due to the axial spacing as had previously been recognised as necessary.

Koch and Smith were able to conclude that the end wall boundary layer, represented by its displacement thickness yielded an efficiency decrease, but this was partly offset by the deficit in the tangential blade force in the boundary layer.

Some engineering considerations

Most of the work considered in this review has been executed under somewhat idealised conditions where the tip gap had a well defined constant value in any particular test. Because of inherent engineering features the tip gap of a turbomachine may not always be constant.

Blade growth due to rotational speed and temperature change automatically reduces the tip gap and this was recognised in early investigations by Lindsey[42], who was forced to relate all of his data to a nominal cold clearance of 0.040 in (0.10 mm). Ewen, Huber and Mitchell[43] were able to quantify the effects of centrifugal growth of a turbine in which the tip gap reduced from 0.014 in (0.35 mm) to 0.010 in (0.025 mm), contributing about 25% of the increase of the 4% efficiency change in changing the operation from a velocity ratio of 0.65 to 0.75. While such an observation diminishes slightly the effect of tip clearances upon turbomachinery operating at design it must also be borne in mind that at reduced speed the tip gap would be increased from such a desirable minimum value and a performance penalty would be incurred in consequence.

An attempt to meet the problem of the change in tip gap due to temperature and rotational effects during an engine cycle has been made by Beitler, Saunders and Wagner[58], who by selective cooling of the turbomachinery casings controlled the tip gaps.

Ovality in turbomachinery casings has traditionally presented problems to the engineer. Simulating a range of in-flight loads upon an operational aircraft engine, Stakolich and Stromberg[59] measured the changes in tip clearance at all compressor and turbine stages through the machine. They concluded that over a period of 2000 flight cycles a 2% change in thrust would occur, mainly due to thermal distortion and erosion of the tips.

An oval casing to a turbomachine produces a time-wise change in the tip gap at any particular blade, but a further problem of non-uniformity is that of blade-to-blade variations in tip clearance. This is an area which has not been specifically addressed, but there is indirect evidence that such variations have an effect upon performance of a turbomachine. Ramachandra[60], investigating a reported noise problem in a particular aircraft cockpit, found a high noise content at 166 Hz. At the compressor operational speed of 10 000 r/min this represented the passing frequency of one blade in a blade row. Now, if the noise were generated due to casing ovality, the dominant frequency would be

$$f_N = \frac{N}{60} \cdot n,$$

where n is the number of blades, but since it was at the frequency

$$f_N = \frac{N}{60},$$

the blade passing frequency, it may be presumed that in the particular case Ramachandra was examining, there was a blade of non-uniform height. Since the noise was generated by some unsteady aerodynamic phenomena it may be concluded from Ramachandra's investigation that a single non-standard gap could have a more general effect upon a machine than at that blade tip and that a noticeable variation in tip aerodynamics, tip to tip, may be present.

Future research

Two objectives are seen in a coherent programme of research into the effects of tip gaps upon turbomachine aerodynamics. The first is the provision to designers of a method to account in design for such effects, both in the assessment of performance loss overall and in modifying design across the blade height to allow for the resulting changes in the flow.

If we consider the tip gaps used in current turbomachinery practice it can be said generally that the penalty on compressor efficiency for the existence of a tip gap is of the order of 5% and that on turbine efficiency 4%. Regaining these efficiency losses by control of the tip aerodynamics could then yield in a gas turbine cycle an improvement in overall efficiency of the order of 5% and this would be most desirable.

To do this, however, the designer needs in his design code a sub-routine that would allow him to modify a design executed using, for example, streamline curvature and blade element design techniques, to include an allowance for the aerodynamic variations due to the tip gap. Such variations, it is seen, could extend over the whole of the blade height and must be related eventually to the tip gap geometry and local conditions.

It remains though to relate the aerodynamic variations to geometry and flow conditions at the tip so that the blade geometry may account suitably for them. Some blade designs have been proposed in which the rotor tip and hub geometry reflect by changes, for instance in local chord and camber, the inlet velocity profile. Beknev[61], Vavra[62], and

Peacock[63] have considered this, but no conclusive data have yet emerged.

The second objective is a base set of data by which geometric modifications, such as casing or tip treatment and design changes radially along a blade, can be referred. Various casing and tip treatment programmes, not reviewed here, have been reported and are in progress. An accurate assessment of the utility of such geometric changes will only be possible when a clear knowledge of performance with plain walls or tips can be quantified.

A coherent programme to meet both the objectives identified above would include an examination of blade rows embedded within a multi-stage machine which, while it would cover the range of geometric parameters encountered in current practice, would also have as controlled variables the annulus boundary layers, the tip gap size and geometry. Measurements would include those necessary to determine overall performance parameters, radial variations of mean flow conditions and the detailed investigation of the tip aerodynamics. For such a programme, instrumentation would necessarily include both that of traditional low response rate and high response rate equipment capable of discriminating the flow conditions at blade passing frequency. Of course, the accurate measurement of the tip gap at operational conditions would be essential.

Conclusions

For turbomachinery, both expansive and compressive, it is possible to make the general statement that the presence of a tip gap reduces performance, an effect which is progressive with increase of gap size. Because of the different relative velocities blade to wall of the compressor and the turbine the detailed mechanics of the flow leading to performance change vary somewhat. It is seen, though, that the aerodynamic effect of the tip gap is not restricted to the tip gap region but can reach over the whole of the blade height, so that adequate allowance for the tip gap in a design procedure must embrace the whole of the blade height.

Research programmes, in which certain components contributing to the overall mechanics of the flow were investigated, were seen[1] to be inadequate in modelling the complex phenomena present as a result of the tip gap. As a consequence, techniques that would permit a process of superposition of various data to synthesise the effects of tip gaps are not anticipated to have validity.

There is disagreement between various workers in allowing for the tip vorticity shed into the working fluid and the prediction method proposed by Lakshminarayana, based upon earlier work by Lakshminarayana and Horlock, while yielding good results in the test cases cited, is based upon an incorrect accountancy of the vorticity in the tip gap. Models proposed by Smith and Koch and Smith, which are based upon measurements from blade rows embedded in turbomachines, are seen to be the most realistic, but do not yield a detailed understanding of the physics of the process and as such do not lead directly to a detailed formulation of the flow.

Data available from tests with various turbines are much more consistent than those from compressors, possibly because in an accelerating flow field, the effect of the varied boundary layers is diminished. In general the loss of performance is a function of gap size and reaction of the turbine stage, some further variation being offered in different tip geometries.

The review indicates that future research would be most profitably executed using representative geometries of turbomachine, particularly in the area of compressor research where diffusing flows heighten the sensitivity of results to the condition of the boundary layer. A further point that emerges is that non-uniform tip gaps, due either to ovalling of the casing or to non-uniformities blade-to-blade, can have an effect upon performance. Since this represents a real engineering consideration this needs detailed investigation.

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